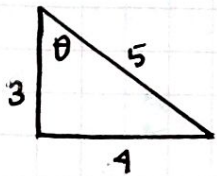


Tugas 2 Matematika 2

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Nama : Mohammad Febryan Khamim
NRP : 5002221072
No. Kelas : 44

1). $\theta = \tan^{-1}\left(\frac{4}{3}\right)$. Nilai eksak $\sin \theta$, $\cos \theta$, $\cot \theta$, $\csc \theta$, $\sec \theta$,



$$\rightarrow \sin \theta = \frac{4}{5} \rightarrow \cot \theta = \frac{3}{4}$$

$$\rightarrow \cos \theta = \frac{3}{5} \rightarrow \csc \theta = \frac{5}{4}$$

$$\rightarrow \sec \theta = \frac{5}{3}$$

2). a. $\frac{d}{dx} \left(\tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right) \right)$

= misal : $u = \left(\frac{1-x}{1+x} \right)^{1/2}$

$$\frac{du}{dx} = \frac{1}{2} \left(\frac{1-x}{1+x} \right)^{1/2} \cdot \frac{-1}{(1+x)^2} dx$$

$$\frac{du}{dx} = - (1+x)^{-3/2} (1-x)^{-1/2}$$

$$\rightarrow \frac{d}{dx} (\tan^{-1} u)$$

$$= \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$= \frac{-1 (1+x)^{-3/2} (1-x)^{-1/2}}{1 + \frac{1-x}{1+x}}$$

b. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$

misal = $u = \tan x$

$$du = \sec^2 x dx$$

$$\int \frac{du}{\sqrt{1-u^2}} = -\sin^{-1} u$$

$$= -\sin^{-1} (\tan x) + C$$

3) a. $\frac{d}{dx} (x^3 \tanh^2(\sqrt{x})) \cdot \dots$

$$u = x^3 \quad u' = 3x^2$$

$$v = \tanh^2(\sqrt{x}) \quad v' = 2 \tanh(\sqrt{x}) \cdot \operatorname{sech}^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = u'v + uv'$$

$$\rightarrow 3x^2 (\tanh^2 \sqrt{x}) + x^3 \left[2 \tanh(\sqrt{x}) \cdot \operatorname{sech}^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \right]$$

$$= 3x^2 (\tanh^2(\sqrt{x})) + x^2 \sqrt{x} \cdot \tanh(\sqrt{x}) \operatorname{sech}^2(\sqrt{x})$$

b. $\int \frac{\sinh 2x}{3 + 5 \cosh x} dx$

misal = $u = 3 + 5 \cosh x$

$$\frac{u-3}{5} = \cosh x$$

$$\sinh x dx = \frac{du}{5}$$

$$du = 5 \sinh x dx$$

$$\rightarrow \int \frac{\sinh 2x}{3 + 5 \cosh x} dx = \int \frac{2 \cdot \cosh x \cdot \sinh x}{3 + 5 \cosh x} dx$$

$$= \int \frac{2 \cdot \left(\frac{u-3}{5} \right)}{u} \frac{du}{5}$$

$$= \int \frac{2u - 6}{5u} \cdot \frac{du}{5}$$

$$= \int \left(\frac{2u}{25u} - \frac{6}{25u} \right) du$$

$$= \frac{2}{25} u - \frac{6}{25} \int \frac{1}{u} du$$

$$= \frac{2}{25} u - \frac{6}{25} \ln u + C$$

$$\rightarrow \frac{2}{25} (3 + 5 \cosh x) - \frac{6}{25} \ln (3 + 5 \cosh x) + C$$

$$= \frac{6}{25} + \frac{2}{5} \cosh x - \frac{6}{25} \ln (3 + 5 \cosh x) + C$$

$$4). d. \frac{d}{dx} \left[(1+x \operatorname{csch}^{-1} x)^{10} \right]$$

$$= 10 (1+x \operatorname{csch}^{-1} x)^9 \cdot \frac{d}{dx} [1+x \operatorname{csch}^{-1} x]$$

$$\begin{aligned} u &= x & v &= \operatorname{csch}^{-1} x \\ u' &= 1 & v' &= -\frac{1}{x\sqrt{x^2+1}} \end{aligned}$$

$$\frac{dy}{dx} = 10 (1+x \operatorname{csch}^{-1} x)^9 \left[\operatorname{csch}^{-1} x + \frac{(-x)}{x\sqrt{x^2+1}} \right]$$

$$= 10 (1+x \operatorname{csch}^{-1} x)^9 \left[\operatorname{csch}^{-1} x - \frac{1}{\sqrt{x^2+1}} \right]$$

$$b. \int \frac{dx}{\sqrt{9x^2-25}} = \frac{1}{3} \int \frac{dx}{\sqrt{x^2 - \left(\frac{5}{3}\right)^2}}$$

$$= \frac{1}{3} \left[\cosh^{-1} \left(\frac{3x}{5} \right) \right]$$

$$5). \int \frac{dx}{\sqrt{x^2-1}} = -\cosh^{-1}(-x) + C$$

$$-y = \cosh x$$

$$x = \cosh^{-1}(-y) = \frac{e^{-y} + e^y}{2}$$

$$\begin{aligned} 2x &= e^{-y} + e^y \\ 2xe^y &= 1 + (e^y)^2 \\ (e^y)^2 - 2x(e^y) + 1 &= 0 \end{aligned}$$

$$e^y, e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{2x \pm \sqrt{4(x^2-1)}}{2}$$

$$= \frac{2x \pm 2\sqrt{x^2-1}}{2}$$

$$= x \pm (\sqrt{x^2-1})$$

ambil (+), karena eksponen tak pernah (-)

$$\begin{aligned} \rightarrow e^y &= x + \sqrt{x^2-1} \\ \ln e^y &= \ln (x + \sqrt{x^2-1}) \\ y &= \ln (x + \sqrt{x^2-1}) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{(x+\sqrt{x^2-1})} \cdot \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right)$$

$$= \frac{1}{(x+\sqrt{x^2-1})} \cdot \left[\frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}} \right]$$

$$= \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx} [\cosh^{-1}(x)] = \frac{1}{\sqrt{x^2-1}} \quad (\text{TERBUKTI})$$